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Technical Notes

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Numerical Simulation of a Plasma Plume Exhaust from an Electrothermal Plasma Thruster

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Nomenclature

h	=	specific enthalpy
I	=	ionization energy
k	=	Boltzmann's constant
k_i, k_r	=	coefficients of ionization and recombination
M	=	Mach number
m	=	mass of particle
n	=	volumetric density (concentration) of particles
p	=	pressure
Q	=	rate of energy exchange
R	=	universal gas constant
r	=	radius
T	=	temperature
t	=	time
$\boldsymbol{v}$	=	velocity vector
υ_x, υ_y, υ_z	=	velocity vector components on the axes x, y, z
α	=	ionization degree
γ	=	specific thermal capacity values ratio
ρ	=	density
σ	=	cross-section of collision

Subscripts

e, i, A	=	electron, ion, and atom characteristics

Superscripts

f	=	elastic processes
ir	=	irradiation
nf	=	inelastic processes

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Introduction

An electrothermal thruster is a space propulsion system that uses electrical energy to heat a low-molecular-weight propellant gas to a high temperature. As in a conventional rocket engine, the heated propellant gas expands in a nozzle to produce the thrust.[1] Resistojet,[2] arcjet,[3] rf-induction arcjet,[4] and microwave electrothermal thruster[1,5–10] are the alternative and competitive electrothermal propulsion systems; each of them has its own advantages and disadvantages.

Numerical models including the effects of the temperature difference between the electron and heavy species have been developed and used to simulate arcjets,[11] magnetoplasmadynamics thrusters,[12] and microwave thrusters.[13] In Ref. 13 a fully coupled calculation, solving both the Navier–Stokes and Maxwell equations, has been performed for the supersonic energy addition section of the two-stage microwave thruster.

At the same time, a wide class of plasma flows with free expansion (supersonic plasma jets from thermal sources) exist, and their investigation does not require use of the Navier–Stokes equations.[14] In Ref. 15, a quasi-neutral plasma in which the electrons and ions behave as ideal gases was considered. Particle collisions were computed using the direct simulation Monte Carlo method. A particle-in-cell method was used to simulate acceleration of the charged particles in self-consistent electric fields. Generally good agreement between experimental and calculated flux profiles was found.

Plasma plume characterization is very important for spacecraft contamination assessment. Although plasma plume exhaust from various plasma thrusters was studied in the past, rf thruster plumes were not well characterized. The goal of the present work is to simulate an expanding plasma jet exhaust from the nozzle of an rf-plasma thruster using the numerical method with a modified finite-difference Godunov scheme based on a mobile grid.[16] The Godunov scheme provides good resolution of the plasmadynamic characteristics through the use of a precise solution of the nonlinear problem of a supersonic plasma moving in the nozzle of the rf-plasma thruster without external magnetic and electrical fields.

Modeling an Expanding Plasma Jet from a Plasma Thruster Nozzle

The steady state expansion of an ideal nonequilibrium plasma jet into vacuum has been the focus of research activity in the past.[6–14] Usually the initial conditions of the problem are the parameters in the nozzle exit plane. In the case of a perfect gas,[14,17,18] the jet outflow is uniquely defined through the Mach number, the inclination angle of the velocity vector to the symmetry axis, and the value of γ in the nozzle exit plane. Outflow of a perfect gas corresponds to outflow of entirely free plasma into vacuum and can be considered as a certain limiting case of the nonequilibrium plasma.

In the general case the equations expressing the laws of conservation of mass, impulse, and energy of a system containing substance and radiation (nonequilibrium in a general case) are given in Refs. 14 and 18. This system of equations models the plasma motion, taking into account energy, radiation, pressure, and radiant heat transfer. Mentioned in Ref. 14, pressure and energy radiation density are smaller than gas-dynamic pressure and internal plasma energy for the relatively low-temperature plasmas of interest. Therefore it is possible to use simple hydrodynamic models to solve many practical problems of applied plasmadynamics. The theory of supersonic plasma jets is mainly based on the hydrodynamically ideal plasma model (nonviscous and non-heat-conductive), which describes the flow in nozzles and jets rather satisfactorily.[14] In addition, one can consider optically thick plasmas where there are no electric and magnetic fields.

The quasi-stationary approximation model for multicomponential plasmas (including electrons, ions, and atoms in the main and excited states) can be considered as a simplified approach. The practical model for such mixtures includes only the processes of ionization and recombination:

$$\frac{\mathrm{d}n_e}{\mathrm{d}t} = k_i n_i n_e - k_r n_e^2 n_i \tag{1}$$

The system of equations[14] can be written as

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x^i} \rho \upsilon^i = 0 \tag{2}$$

$$\frac{\partial n_e}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x^i} n_e \upsilon^i = k_i n_i n_e - k_r n_e^2 n_i \tag{3}$$

$$\frac{\partial \rho \upsilon_i^i}{\partial t} + \sum_{j=1}^{3} \frac{\partial}{\partial x^j} (\rho \upsilon^i \upsilon^j + p \delta^{i,j}) = 0 \tag{4}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}t} - \frac{Q^{\mathrm{ir}}}{\rho} \tag{5}$$

$$\frac{5}{2}\frac{k\alpha\rho}{m_A}\frac{\mathrm{d}T_e}{\mathrm{d}t} = \frac{\mathrm{d}p}{\mathrm{d}t} + Q_e^f + Q_e^{\mathrm{nf}} - Q_e^{\mathrm{ir}} \tag{6}$$

$$p = \rho R(T + \alpha T_e), \qquad p_e = \alpha \rho R T_e \tag{7}$$

Here enthalpy is

$$h = \tfrac{5}{2} R(T + \alpha T_e) + (\alpha/m_A) I \tag{7a}$$

Ionization degree and gas constant are

$$\alpha = n_e/(n_e + n_A) \approx m_A n_e/\rho, \qquad R = k/m_A$$

Coefficients of ionization and recombination are

$$k_i = k_i(n_e, T_e), \qquad k_r = k_r(n_e, T_e)$$

Rates of energy exchange by the elastic processes (electron–atom and coulomb) are

$$Q_e^f = Q_{\mathrm{eA}}^f + Q_{\mathrm{ei}}^f$$

$$Q_{\mathrm{eA}}^f = \left(\tfrac{3}{2}\right) n_e n_A [8kT_e/(\pi m_e)]^{\frac{1}{2}} \times [(T - T_e)/T_e] k T_e (m_e/m_A) \langle \sigma_{\mathrm{eA}} \rangle$$

$$Q_{\mathrm{ei}}^f = n_e n_i \left(e^4 \big/ m_i\right) [8\pi m_e/(kT_e)]^{\frac{1}{2}} [(T - T_e)/T_e]$$

For the optically thick plasma the equations for Q_e^{nf}, Q^{ir}, and Q_e^{ir} can be presented as[14]

$$Q_e^{\mathrm{nf}} = -\left(I + \frac{5}{2} kT_e\right)\frac{\mathrm{d}n_e}{\mathrm{d}t}, \qquad Q^{\mathrm{ir}} = 0, \qquad Q_e^{\mathrm{ir}} = 0$$

The state equation (2), motion equation (4), and energy equation (5) are written for a plasma as a whole. There is no kinetic energy of electrons, because its value is small compared to the internal energy magnitude in the energy equation for electrons (6).

Equation (3) can be rewritten in the form

$$\frac{\partial \rho\alpha}{\partial t} + \sum_{i=1}^{3} \frac{\partial}{\partial x^i} \rho\alpha\upsilon^i = k_i \frac{\rho^2}{m_A}\alpha(1-\alpha) - k_r \frac{\rho^3 \alpha^3}{m_A^2} \tag{3a}$$

Analytical and numerical analyses[14,19] show that for a wide range of parameters the energy of electrons is defined mainly by the losses through elastic collisions with ions and return of the effective recombination energy into the electron gas. Thus

$$Q_e^f + Q_e^{\mathrm{nf}} = 0 \tag{8}$$

In these cases the energy equation for the electrons can be greatly simplified. Then one can use the dependence $k_r = k_r(T_e)$ of the recombination process to present the electron energy equations in the balance form. As shown in Ref. (19), the electron energy equation has the balanced form $T_e^3(T_e - T) = Cn_e$ for a rather wide range of parameters in a supersonic source. It is important to notice that under the conditions considered here, the values n_e and T_e are uniquely defined by the balance equation and do not depend on the initial parameters. This assertion is confirmed by the experimental data.[14]

The system equations (2)–(7) can be solved by the numerical method.

Numerical Calculation Methods

The established numerical method[20] is based on the nonstationary plasma dynamics equation and solving for the steady-state flow as a limiting case when $t \to \infty$. In the final volume frames method,[21] the differential equations describing a flow of nonviscous non-heat-conductive nonequilibrium gas are written as the integral conservation law. The Cartesian coordinate system equations for the two-dimensional plasma flow can be presented in vector form as[21]

$$\frac{\partial}{\partial t}\int_{\Omega} F \,\mathrm{d}\Omega + \int_{\sigma} \bar{A}\bar{n}\,\mathrm{d}\sigma = \Phi \tag{9}$$

where

$$F = \begin{bmatrix} \rho \\ \rho\alpha \\ \rho u \\ \rho\upsilon \\ \rho E \end{bmatrix}, \qquad \vec{A} = \begin{bmatrix} \rho(\bar{q} - \bar{\lambda}) \\ \rho\alpha(\bar{q} - \bar{\lambda}) \\ \rho u(\bar{q} - \bar{\lambda}) + p\bar{i}_x \\ \rho\upsilon(\bar{q} - \bar{\lambda}) + p\bar{i}_y \\ \rho E(\bar{q} - \bar{\lambda}) + p\bar{q} \end{bmatrix}, \qquad \Phi = \begin{bmatrix} 0 \\ \phi_e \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Here $\bar{i}_x$, $\bar{i}_y$ are the unit vectors of the Cartesian coordinate system; F are conservative vector variables; $\bar{A}$ are conservative flow vector variables; σ is the limiting surface of a certain volumetric element, which has an external normal and is moving with the velocity vector of gas flow, $\bar{\lambda}$; $\bar{q} = u\bar{i}_x + \upsilon\bar{i}_y$ is the velocity vector of gas flow;

$$E = \tfrac{3}{2}(k/m_A)(T + T_e) + \alpha I/m_A + q^2/2$$

is the specific total energy; and ϕ_e is the mass velocity formed by a charged component as a result of elementary processes (ionization and recombination). This mass velocity, added to the right part of the continuity equation, is related to the charged component, and by calculating the ideal and two-atomic gas it is found to be equal to zero. It is a scalar value. What it represents can be seen in Eq. (3). The difference between the present formulation and the regular one is that the cell velocity $\bar{\lambda}$ is included in the main equation (10) and each volume element depends on time. According to the authors' opinion, this provides a better flow parameter calculation procedure through the cell side. The equations have advantages for numerical calculations, which give the opportunity to operate with unspecified cells on mobile sides. It is convenient in calculating areas with free boundaries, as far as grid cells configuration correction during the calculation is concerned.

The considered flow area is located among the surfaces of the body (including the edge of the nozzle), the change surface, any surface through which plasma outflows the area and the surface of symmetry. The nozzle is limited by shock waves at finite pressure in the freely expanding environment area and the outflow into the vacuum extent of the freely expanding area becomes infinite. Two families of lines into the finite elementary volumes (cells) divide the numerical simulation results. Plasma parameters for a fixed time t inside each cell are constant, averaged on the volume, and change only in going from one cell to another. One can define the position of any cell in the limits of the plotted differential grid with a number of indices k, m ($k = 0, \ldots, K$; $m = 0, \ldots, M$).

The equations (9) are solved for each elementary cell. The gas-dynamic set parameters in all the cells at time t are a well-known solution in the layer with index n. The parameters in the layer $n + 1$ (where t is time spacing) are calculated by application of explicit differential approximations by Eqs. (9) and the frames of the method of finite volumes,

$$F^{n+1}_{k-\frac{1}{2},m-\frac{1}{2}} = L(\Delta t) \prod_{\beta=1}^{\varepsilon} L_\alpha(\tau) L_c(\Delta t) F^{n}_{k-\frac{1}{2},m-\frac{1}{2}} \qquad (11)$$

Here the upper indices indicate the numbers of time layers, and the lower ones indicate the numbers of calculation grids. The values with integral lower indices are defined in the calculated cell. The calculation method for the flow $\bar{A}$ is determined by the choice of a differential scheme. The modified Godunov scheme is used in this paper. Operator $L(\Delta t)$ defines the parameter values at the time $t^n + \Delta t$, and according to their previous values defined at time t^n under the condition that a grid is fixed and $\Phi = 0$. As far as the practical solution of the equations ($\Phi \neq 0$) requires a time spacing τ smaller than the spacing Δt, the produced term is separated from the others and the operator $L_\alpha(\tau)$ is repeated for each cell as many times as needed till the sum of consecutive spacings $\sum \tau$ becomes equal to the final time value t. The effect of cell motion is taken into consideration through the grid operator $L_c(\Delta t)$.

Calculation of motion of the external boundary of the area is realized similarly to that described in Ref. (19). Motion of the external boundary of the area is discontinued when it reaches the conditional boundary area of continuous flow. It is defined according to the methods of Ref. (19). After the setting of a new position for the movable boundaries of the calculation area, the grid generation is made in accord with a chosen law of node distribution (as realized in Refs. 16, 20, and 22).

Boundary Conditions

Let us consider the problem of the two-dimensional stationary expansion of the nonequilibrium plasma outflow from a conic nozzle with semiopening angle ~11 deg and radius $r_a = 1$ cm (a similar problem was considered in Ref. 14 through the characteristics method).

We assume that the edge of the nozzle flow geometry corresponds to the flow in the source with a pole in the point of intersection between the generating line of a nozzle and the symmetry axis x. The calculation diagram area is shown in Fig. 1.

As a working example we consider the rf plasma thruster developed at Kharkov Aviation Institute. The thruster has the following parameters: resonator chamber length 0.3 m and diameter 0.06 m; frequency 120 MHz; power from 0.5 to 1 kW. The experimental data resulted in the following parameters at the exit plane (surface aa'): $M = v/a = 2.5$; $v = 1.4 \times 10^4$ M/c; $T = 2000$ K; $\rho = 5 \times 10^{-5}$ kg/m^3; $p_a = 830$ Pa. The values α and T_e on the initial surface are determined through Eqs. (7) and (7a). The dimensions of the calculation grid changed during the calculation fulfillment from 10×80 to 100×80.

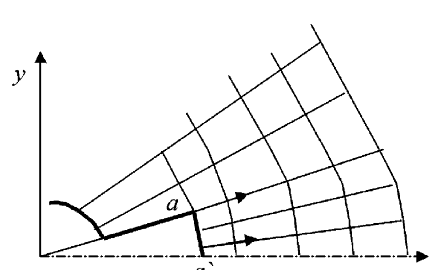

Fig. 1 The calculation area scheme.

Simulation Results

The Mach number and pressure calculation results at the outflow of an ideal perfect monatomic gas ($\gamma = 1.67$) are shown in Figs. 2 and 3.

The x and y coordinates are related to r_a: $\bar{x} = x/r_a$ and $\bar{y} = y/r_a$. The pressure parameters are related to the pressure value on the surface through aa': $\bar{p} = p/p_a$. We show (Fig. 4) the Mach number calculation values on the axis of the hydrogen plasma jet, which are compared to Mach number on the jet axes of the ideal perfect monatomic ($\gamma = 1.67$) and diatomic ($\gamma = 1.4$) gases.

Energy release in recombination expanding plasma leads to the M flow area reduction and a more intensive turn from the axis symmetry compared to that for the entirely frozen outflow corresponding to $\gamma = 1.67$. In this case, the flow geometry relaxation processes and the density field velocity influence are not greater than those explained by the moderate-ionization-energy transition into the translation degrees of freedom. The variation of electron density along the axis x is presented in Fig. 5.

The ionization energy on the edge of the nozzle is nearly 20% of the total enthalpy for the assumed parameters. However, the release of a small portion of the accumulated energy

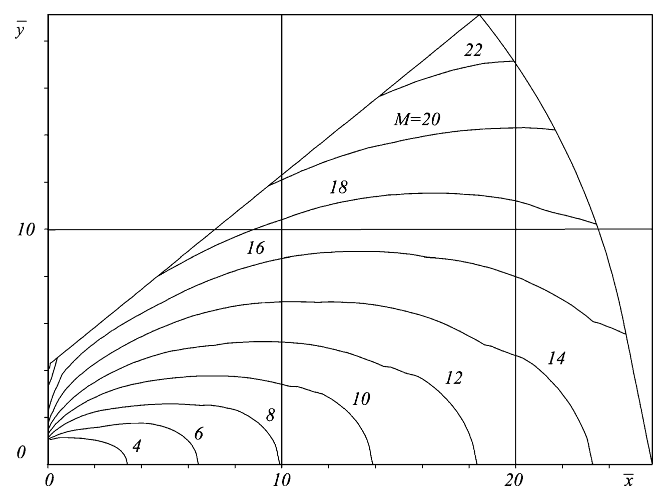

Fig. 2 Equal level lines of Mach numbers in an axially symmetric jet of an ideal perfect monatomic ($\gamma = 1.67$) gas, which outflows from a round nozzle into a vacuum.

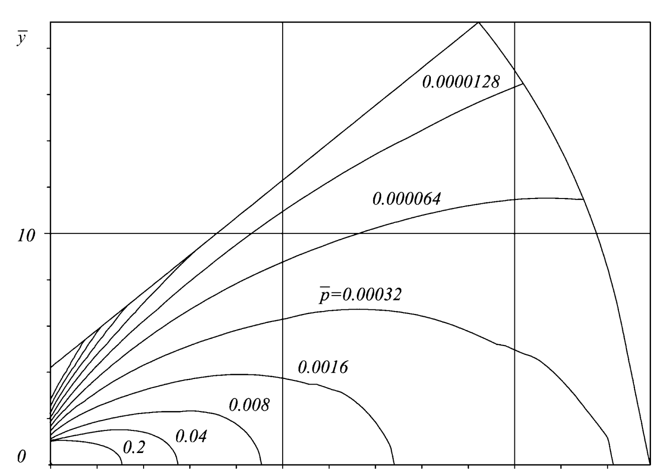

Fig. 3 Equal level lines of pressure in an axially symmetric jet of an ideal perfect monatomic ($\gamma = 1.67$) gas, which outflows from a round nozzle into a vacuum.

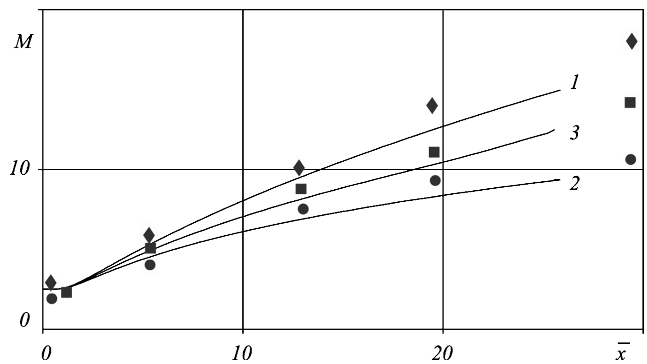

Fig. 4 Comparison of Mach number change for hydrogen plasma axes jet, which outflows into the vacuum from the rf thruster, and the jets of the ideal perfect monatomic and diatomic gases. Monatomic gas: 1, present work, ◆, Ref. 14. Diatomic gas: 2, present work, ●, Ref. 14. Plasma: 3, present work, ■, Ref. 14.

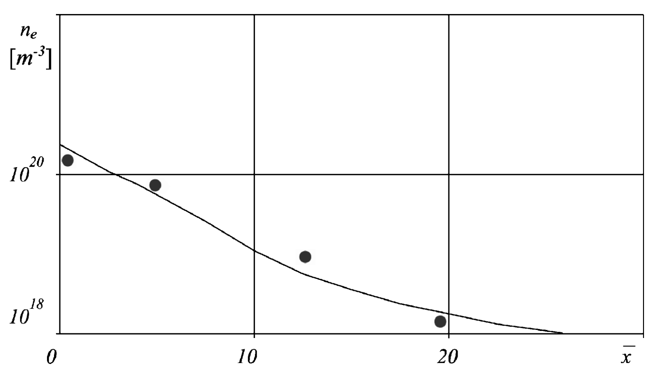

Fig. 5 The electron density variation along the axis *x*: ——, present work, ●, Ref. 14.

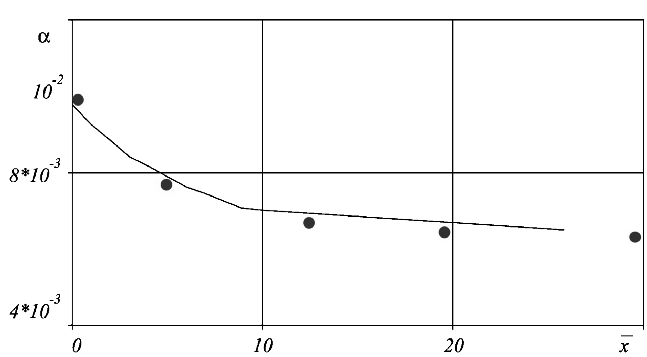

Fig. 6 The ionization variation degree along the axis *x*: ——, present work, ●, Ref. 14.

only occurs as a result of a rapid ionization frozen degree (Fig. 6).

Conclusions

In this paper the free expansion of ideal nonequilibrium plasma-stream exhaust from the nozzle of the electrothermal thruster was considered. The outflow media consist of ions, electrons, and neutrals (atoms and molecules). The nonstationary plasma dynamics equations were solved by modifying the finite difference Godunov scheme based on a mobile grid. The calculations for the auxiliary outflow of the one-velocity, three-component, and two-temperature plasma from a round nozzle into the vacuum were performed. As a result the Mach number lines and pressure distribution in the axially symmetric jet for the ideal perfect monatomic plasma were obtained. Our calculations agree well with similar data[14] obtained through the method of characteristics.

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